A Design Method for Hydraulic Positioning Systems

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ABSTRACT

This paper presents a design method for hydraulic positioning systems. The method takes account of the positioning system performance requirements, the load forces that should be applied, and the hydraulic power supply system characteristics. In this regard, the steps to design a system are shown for a real case, where a proportional valve and a hydraulic cylinder are used for validation purposes.

INTRODUCTION

Position control using hydraulic systems is widely applied in several engineering and industrial fields, including aerospace, vehicles, and speed governors. However, their design and assembly are not simple tasks since it is necessary to observe their behavior according to control theory. Each application has specific static and dynamic requirements that must be fulfilled under loadings which are not well known by the designer. These difficulties have led to the reuse of previous designs for the construction of new equipment, which requires changes or adjustments to some components in order to achieve the desired behavior. This results in increased development cost and time. In order to overcome these constraints, a comprehensive view of the design problem is necessary to create procedures which assure the correct hydraulic component sizing, giving the designer complete domain over the technical decisions during the design process.

In this regard, a method was developed at LASHIP¹ to organize the design effort into three main steps: static and dynamic sizing of the system, conversion of cataloged data and dynamic behavior study.

The first step calculates the minimum requirements for the hydraulic cylinder and the proportional valve that comprise the actuator system. This is carried out based on simple performance requirements, such as settling time, overshoot and maximum load.

The second step compares the required specifications of the hydraulic components to a database of components available on the market.

Subsequently, the system composed of the chosen components is studied with the help of a computer simulation model in the third step, in order to analyze the system performance under detailed operational conditions in terms of piston friction, type of controller, valve dead-zone, load changing, etc.

A LabVIEW® application²⁾ was developed to support the first two steps. The third step was implemented in a MATLAB®/Simulink® model designed to deal with the type of actuator employed.

The case studied in this paper employs a four-way spool valve and a symmetric cylinder. However, the method has also been developed to handle systems employing a four-way spool valve and a differential cylinder set or a three-way differential spool valve and an asymmetric cylinder set.

THE DESIGN METHOD

The design of mechatronic systems can be structured into four main phases according to the technical systems design methodology presented by PAHL and BEITZ³⁾. The design phases are: clarification of the task, conceptual design, embodiment design, and detail design. As shown in

Figure 1, they are executed in a preferential sequence where the results from each phase can be used to review earlier decisions. Likewise, each phase comprises steps and activities to be carried out by the designer in order to complete the design process.



Figure 1 – Product design process⁴⁾

The design method presented in this paper is part of the embodiment design phase of a system, and is performed according to Figure 2. Before this method is implemented, both clarification of the task and conceptual design phases must be carried out in order to generate the first version of the requirements and the concept, which are necessary for the execution of the embodiment activities. This information might come from professionals in other areas, who understand the process or the equipment where the system will be installed.



Figure 2 – Embodiment design phase for a hydraulic positioning system¹⁾

Considering that the aim of this method is to manage the hydraulic positioning system design, its conception depends on the type of hydraulic circuit chosen to solve a specific problem. One can observe that there are three basic configurations that have been widely applied in industry: (a) Symmetric proportional four-way valve and symmetric double effect cylinder; (b) Asymmetric proportional four-way valve and asymmetric double effect cylinder; and (c) Proportional three-way valve and asymmetric single effect cylinder. Figure 3 shows these hydraulic circuits including the position transducer and the controller that are necessary to create a hydraulic position control system.



a) A four-way spool valve and a symmetric cylinder set



b) A four-way spool valve and an asymmetric cylinder set



c) A three-way spool valve and an asymmetric cylinder

Figure 3 – Three common configurations for hydraulic positioning systems¹⁾

In the following sections the three steps of the embodiment design are detailed for configuration (a), that is, a symmetric four-way valve with a symmetric cylinder. Results related to the other two arrangements can be found in FURST¹.

STEP 1 – STATIC AND DYNAMIC SIZING – this step considers the dynamic and static requirements of the system, such as maximum displacement, settling time, forces, overshoot, among others, and uses the mathematical expressions derived from the dynamic model of the system. As a result, specifications for a valve and a cylinder are achieved. These components must be selected simultaneously, since their parameters are interdependent. The activities can be executed in the sequence shown in Figure 4.



Figure 4 – Static and Dynamic Sizing¹⁾

System undamped natural frequency (Activity 1) – This activity begins by analyzing the global system behavior, instead of each component that comprises it. If the system can afford overshoot, it will be treated as a second-order system with a damping ratio coefficient $\zeta^{SYS} = 0.7$. If having an overshoot is not feasible, a second-order system with $\zeta^{SYS} = 1$ will be used. The general time responses of such systems are shown in Figure 5.

With this information and the desired system settling time t_s^{SYS} (Figure 6) it is possible to calculate the system undamped natural frequency ω_n^{SYS} as represented in Figure 7 using equation (1) or (2).



Figure 5 - Time response of second-order systems



Figure 6 - Settling time (2% criteria)



Figure 7 - Activity 1 flowchart

$$\omega_n^{SYS} = \frac{5,7}{t_s^{SYS}}$$
 for $\zeta^{SYS} = 0.7$ (1)

$$\omega_n^{SYS} = \frac{6}{t_s^{SYS}} \text{ for } \zeta^{SYS} = 1 \quad (2)$$

<u>Cylinder speed and acceleration (Activity 2)</u> - Based on the information obtained from activity 1 and on the desired steady state displacement for the positioning system, the maximum speed v_{max}^{c} and acceleration a_{max}^{C} can be calculated using equations (3) to (6) according to Figure 8.





If $\zeta^{SYS} = 0.7$

$$a_{\max}^{C} = x_{\max}^{C} \cdot \omega_{n}^{SYS^{2}} \quad (3)$$

$$v_{\max}^{C} = 0.46 \cdot x_{\max}^{C} \cdot \omega_{n}^{SYS} \quad (4)$$

If $\zeta^{SYS} = 1$

$$a_{\max}^{C} = x_{\max}^{C} \cdot \omega_{n}^{SYS^{2}} \quad (5)$$
$$v_{\max}^{C} = 0.37 \cdot x_{\max}^{C} \cdot \omega_{n}^{SYS} \quad (6)$$

Load pressure (Activity 3) - For a system composed of a symmetric proportional four-way valve and a symmetric cylinder, the load pressure is defined as:

$$p_C = p_A - p_B \quad (7)$$

The maximum load pressure during the system operation must be estimated under the following conditions: maximum displacement, maximum speed and maximum acceleration. In order to do this, the load stiffness K, viscous friction coefficient B and mass M_C must be available.

<u>Cylinder area (Activity 4)</u> - MERRIT⁵⁾ shows that the maximum output power for this positioning system occurs at the load pressure calculated by:

$$p_{CP\max} = \frac{2}{3} \cdot p_S \quad (8)$$

where the supply pressure p_s is a design requirement defined in a previous phase.

It is important to note that the expected load pressure should not be higher than the load pressure at the maximum power transfer point. If the load is increased above this limit, the pressure drop across the valve approaches zero, and the hydraulic actuator tends to stall. Considering this, the cylinder area is calculated as shown in

Figure 9:



Figure 9 - Activities 3 and 4 flowchart

The cylinder area calculated is the annulus area. Thus, the maximum load pressure must be calculated under three operational conditions:

• When the cylinder is positioned at the maximum desired position:

$$A^{\mathsf{C}} \cdot p_{C} = K \cdot x_{\max}^{\mathsf{C}} + F_{U} \quad (9)$$

where F_U is the useful force applied to the load and K is the load stiffness.

• When the cylinder is at its top speed. $A^{C} \cdot p_{C} = B_{t} \cdot v_{max}^{C} + F_{U} \quad (10)$

where B is the viscous friction coefficient.

• When the cylinder is at its maximum acceleration. $A^{C} \cdot p_{C} = M_{t} \cdot a_{max}^{C} + F_{U} \quad (11)$

where:

$$M_{t} = M_{e} + M_{C} + M_{f}$$
 (12)

where M_{t} is the total mass accelerated by the actuator, composed of the mass of the moving parts of the

cylinder M_{e} , the load mass $M_{C}\,$ and the fluid effective mass M_{ℓ} .

 $\boldsymbol{M}_{\boldsymbol{f}}\,$ can be calculated by the expression:

$$M_{f} = M_{f}^{C} + \sum_{1}^{n} M_{i} \cdot \left(\frac{A^{C}}{A_{i}}\right)^{2}$$
 (13)

where M_f^C is the fluid mass inside the cylinder chamber, M_i is the fluid mass inside the pipeline i, A_i is the area of the section of pipeline i, and n is the total number of pipelines with different sections between the valve and the cylinder chamber⁶. When these pipelines are long and narrow, the effective fluid mass can be considerably larger than the fluid mass.

After the required area has been defined, a commercially available cylinder must be chosen from cataloged data with an area equal to or greater than the area calculated in this activity. On the other hand, if a commercial cylinder is already available, the equations above are solved not to define the area, but to find $p_{C\rm max}$, in order to verify whether the supply pressure is high enough so that the 2/3 value is not exceeded. If the supply pressure is too low, the cylinder area must be increased, and a new commercially available cylinder has to be selected, or the supply pressure p_S can be redefined at this stage to make the use of an available cylinder feasible.

<u>Cylinder maximum flow rate (Activity 5)</u> - Subsequently, when the cylinder area is defined, the designer is able to determine the maximum flow rate of the cylinder based on its maximum speed using the equation below:

$$q_{VC\,\max} = A^C \cdot v_{\max}^C \quad (14)$$

<u>Cylinder undamped natural frequency (Activity 6)</u> - The next activity is to calculate the natural frequency of the cylinder ω_n^C when associated with the load by the following equation:

$$\omega_{n_{\min}}^{C} = \left(\frac{4 \cdot \beta e \cdot A^{C^{2}}}{M_{t} \cdot V_{t}^{C}}\right) \quad (15)$$

This value should be at least equal to the undamped natural frequency required for the system (Activity 1). This is a physical limit that determines whether the cylinder and its load have the necessary bandwidth to yield the desired time response. <u>Valve undamped natural frequency (Activity 7)</u> - The valve natural frequency ω_n^V can be obtained according to the flowchart in Figure 10:



Figure 10 - Activity 7 flowchart

The valve natural frequency must be at least three times the smaller natural frequency to avoid further slowing of the system time response.

<u>Valve settling time (Activity 8)</u> – Once the valve ω_n^V value is defined, its settling time t_s^V can be found by means of the equation below:

$$t_{s}^{V} = \frac{5,7}{\omega_{n}^{V}}$$
 (16)

Finally, with the data now available, a commercial valve must be chosen from cataloged data.

STEP 2 – DATA CONVERSION FROM DATASHEETS AND CATALOGS – A valve with similar characteristics to the one specified in Step 1 must be found among those available on the market. However, each manufacturer presents the technical data in specific ways that require careful analysis by the user. To overcome this problem, the flow rate coefficient, Kvwas defined¹, which can be easily calculated from the catalogs. This single parameter allows the designer to compare the flow rate capability of the valves and their suitability according to the specifications obtained in Step 1.

Hence, the flow rate coefficient is used in Step 2 as shown in Figure 11.

Pressure drop at valve at maximum flow rate (Activity 1) – Using the maximum load pressure at the maximum speed obtained in Step 1 – Activity 4, one can calculate the total pressure drop at the valve as:

$$\Delta p_t = p_s - \left| p_{C \max} \right| \quad (17)$$

The type of presentation must be identified to calculate the Kv value correctly.

Flow rate coefficient (Activity 2) - Considering the maximum flow rate obtained in Step1 - Activity 5 and

the total pressure drop above, it is possible to determine the flow rate coefficient through Equation (18).

$$Kv = \frac{q_{VC \max}}{\sqrt{\Delta p_t}} \quad (18)$$

This equation establishes the minimum value that will guarantee the condition of maximum cylinder speed.



Figure 11 – Data conversion from datasheets and catalogs¹⁾

<u>Catalog flow rate coefficient (Activity 3)</u> – To calculate the commercial valves' Kv_{cat} the designer may extract a point on the valve flow rate versus pressure drop curve or the nominal flow rate (q_{Vn}) at a nominal pressure drop from the catalog. However, considering that some manufacturers present the flow rate data as pressure drop per notch (partial pressure drop), while others present this in terms of total pressure drop at the valve, the catalog flow rate coefficient must be calculated by:

For total pressure drop:

$$Kv_{cat} = \frac{q_{Vn}}{\sqrt{\Delta p_{tn}}} \quad (19)$$

For partial pressure drop:

$$Kv_{cat} = \frac{q_{Vn}}{\sqrt{2\Delta p_{pn}}} \quad (20)$$

Special attention must be paid to the units used. A common set of units must be chosen by the designer

and all data from the different catalogs must be converted to these units prior to the ${}^{K\!v_{cat}}$ calculation.

The chosen valve must have a coefficient similar to or greater than that specified in Activity 2, as shown below:

$$Kv_{cat} \ge Kv$$
 (21)

<u>Maximum flow rate at the valve (Activity 4)</u> - The effective maximum flow rate is a consequence of the valve chosen and is determined as:

$$q_{VC\max} = K v_{cat} \cdot \sqrt{\Delta p_t} \quad (22)$$

However, attention must be paid to the curve of the power limits of the valve, where the operational condition established by the coordinate ($q_{VC\max}, \Delta p_t$) must be inside the limits indicated by the curves, as in the cases shown in Figure 12. Observe that these graphs consider the pressure drop per notch Δp_{p} .



Figure 12 – Valve power limit: pressure drop per notch x flow rate $^{1)}$

The black curves $U_{c\min}$, U_{c4} , U_{c5} and $U_{c\max}$ in Figure 12 represent the spool opening for different voltages at the solenoid. The red curve represents the valve limit above which the solenoid will not be able to keep the valve open. The valve application range is inside the blue area.

STEP 3 – DYNAMIC BEHAVIOR STUDY – After finding a commercially available cylinder and proportional valve, a simulation is recommended in order to validate the design. However, this step may be ignored when a system with similar requirements has been previously implemented by the designer with success. In this case, any necessary adjustments can be carried out to the system controller during commissioning. A reliable computer model must be available to carry out a design validation simulation. The model used in this paper was developed using the Simulink® Software present in MathWorks MATLAB®. Models built under Imagine AMESIM® were also tested and gave successful results.

The model should simulate the behavior of components in an environment as similar as possible to that where the system will operate. The controller, position transducers, hydraulic power unit, hydraulic fluid, load and other relevant environmental details must be considered in order to assess the system behavior under such conditions.

Without a doubt, simulation prevents the designer from spending resources on a system that might not behave as expected due to some other component in the system, or that may be hard to tune due to the controller used on the closed loop. It also saves time, since it is not necessary to assemble the system to test different sets of components to define which combination best meets the performance specifications.

However, the model must be validated before being used to make sure the results are reliable. Also, the designer must be well aware of the model's capabilities and limitations, in order to avoid making decisions based on a simulation that may not be accurate under certain conditions. Once the model for one component has proven to be accurate, it can be combined with models for other components in order to assemble a system model.

Finally, if the performance specifications are not met during the simulation or during the workbench or field tests, even after tuning the controller and other components of the system, it may be necessary to go back to Steps 1 and 2 to redesign the positioning system.

In other words, the dynamic behavior study is well represented by Figure 13.



Figure 13 – Dynamic behavior study¹⁾

THE DESIGN AND CONTROL SOFTWARE

With the aim of organizing the activities in the method and making the calculation automatically, a LabVIEW® application was built up in LASHIP⁴ (Figure 14). The software implements all tasks in the described method and also has a help menu to aid the user during the design. Moreover, the three positioning system configurations shown in Figure 3 are implemented.

Finally, a database with data from several manufacturers' catalogs is available in the Step 2 module of the software.



Figure 14 – Design application interface: positioning system type selection screen²⁾

In addition to the design interfaces, a control interface has been implemented to control components assembled in LASHIP's PHP (Proportional Hydraulic Platform - Figure 15). The PHP includes a hydraulic power unit (HPU) controlled by a programmable logic controller (PLC), a VXI data acquisition system, pressure and displacement transducers, a PC running LabVIEW® and MATLAB®, a workplace, signal routing equipment, pressure reducing valves and other components to simulate the environments where the systems being tested will be used¹.



Figure 15 – Proportional Hydraulic Platform

DESIGN METHOD EVALUATION

The PHP has a pneumatic load system that is able to provide several load levels. In order to evaluate the design method presented in this paper, a load was set in the PHP and a positioning system was designed making use of the application in LabVIEW®. The performance specifications and the load characteristics of this positioning system are summarized in Table 1, which also shows the results given by the application that implements Step 1. The cells in gray are calculated by the software, and the cells in white are entered by the user. During Step 1, a Bosch Rexroth cylinder with an 18 mm rod diameter and a 25 mm piston diameter was selected. The annulus area is 2.364 cm², which is close to that suggested by the method. Once this data was inputted into the software, the valve parameters were calculated.

Table 1 – Requirements for the positioning system and results from Step 1^{7}

Requirements		Results	
t_s^{SYS} [ms]	450	$v^C \max[m/s]$	0.39
$x^{C} \max[mm]$	80	$a^{C} \max[m/s^{2}]$	14.22
Overshoot?	No	$p_c \max [Pa]$	1.27e6
ω_n^{SYS} [rad /s]	12.69	$q_V^C \max[lpm]$	5.56
$p_{s}[Pa]$	4.50e6	ω_n^C [rad / s]	190.5

¹ The PHP was designed and built by LASHIP through a project sponsored by NFPA, Bosch Rexroth and Hydac.

$A^{C}[cm^{2}]$	2.36	t_s^{V} [ms]	150
F_u [N]	708	$\omega_n^V \geq 3 \cdot \omega_n^{SYS}$	
$B[N \cdot s / m]$	340	ω_n^{V} [rad /s]	38.10
$M_t[kg]$	5.50	$Kv [m^3 / s\sqrt{Pa}]$	7.56e-8

After Step 1 had been completed, a commercial valve was selected from the valves available on the PHP. The data given in the catalogs was entered into the software which then implements Step 2, as summarized in Table 2.

The Kv_{cat} and the ω_n^V values must be equal to or greater than the calculated values. Since the Bosch Rexroth valve model 0811 404 038 is the closest to that suggested by the method, with the same Kv_{cat} and a ω_n^V much larger than the required value, it was chosen to assemble the positioning system.

Before the actual assembly of the system, a model representing the positioning system, the PID controller, position transducer and pneumatic load was built in MATLAB®'s Simulink®⁷. This model is comprised of blocks based on transfer functions that represent each of the components of the system assembled in a closed loop. The model uses parameters available in the manufacturers' catalogs for each component as well as in the data generated by the design method. Each block in Figure 16 stands for a sub-model that represents a component.

Table 2 – Valv	es available in	the PHP	Database
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Description	Maker\ Model	Kv _{cat}	ω_n^V
Symmetrical Proportional Directional Valve (Servo	Bosch 0 811 404	7.56e-8	439.60
solenoid)	601		
Symmetrical Proportional	Bosch		
Directional Valve (Servo solenoid)	0 811 404 038	1.51e-7	439.60
Symmetrical Proportional Directional Valve	Bosch 0 811 404 119	4.61e-7	166.61
Differential Proportional Directional Valve	Bosch Rexroth 4WREE06 E1-08-2X	1.33e-7	157.00
Differential Pilot Operated 2-stage Proportional Directional Valve	Rexroth 4WRZE10 E1-85-7X	1.42e-6	71.40
Symmetrical Proportional Directional Valve (Servo	Hydrus HDR01DP	2.25e-7	199.70
Symmetrical Servovalve	Moog 760C263A	2.33e-7	219.80



Figure 16 - Simulink® model for the positioning system

The pneumatic load and the cylinder blocks were designed according to experiments executed on the PHP in order to assess the friction forces involved. The pneumatic load behaves as in Figure 17, and the viscous friction coefficient in the hydraulic cylinder varies, as shown in Figure 18.



Figure 17 – Pneumatic load x cylinder speed⁷



Viscous Friction Coefficient x Speed

Figure 18 – Viscous friction coefficient f_V x cylinder speed⁷⁾

The controller implemented in MATLAB is the same PI (Proportional – Integral) controller implemented by the LabVIEW® in the PHP. The components selected by the method behaved very well during the simulation, satisfying all requirements established. The controller was tuned in the model, and then the positioning system was assembled.

The positioning system was tested under the same conditions simulated in the model and the results were very similar, as seen in Figure 19. Otherwise, aiming to analyze the method effectiveness, the system was tested under different loads (Figure 20).

As one can see, the system was faster and had an overshoot when tested under a smaller load (515 N plus friction forces). This can be fixed by adjusting the controller with smaller gains. On the other hand, the system was not able to meet the settling time specification with higher loads (927 N and 970 N plus friction forces).







Figure 20 – Step response to different load levels⁷

In addition, the valve specified was replaced by a valve with twice the necessary Kv. The system met all specifications after the controller was tuned to control the new valve. However, the simulations showed that if a valve with a Kv value three times higher than the required value is used, the system would be considerably faster if smaller gains on the controller are applied, and unstable if the same controller tuning is used. Also, if the controller is tuned in such a way that the system would have a settling time similar to the specified time, the gains would be so low that the system would be slower than systems with smaller valves when close to the set point.

In summary, the simulated performance of the system for different valves under the same controller is depicted in Figure 21.



Figure 21 – Step response using valves with Kv = 1.43, 1.78, 2.86, 4.29

CONCLUSIONS

The case results demonstrated that the method is a useful tool for hydraulic positioning system designers. The components selected were adequate for the task they were expected to perform. Moreover, the simulation successfully aided the designer in tuning the controller and assessing the performance of different components before the system was assembled, with no risks to equipment or personnel.

Since the same performance can be obtained with different sets of components with the adjustment of the controller, the choice might be made based on dimensional, reliability or cost factors. However, there are limits to the adjustments that can be made on the controller, and that is where the simulation is useful.

Furthermore, other considerations must be taken into account to make the best use of the method.

The specifications must be as precise as possible. Whether the designer wants to reach at least or exactly the specified settling time will make a big difference in valve selection. In the second situation, a valve with a Kv much higher than suggested by the method will not be suitable.

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DEFINITIONS, ACRONYMS, ABBREVIATIONS

- *a* : Acceleration $[m/s^2]$
- A : Area $[m^2]$
- *B* : Viscous friction coefficient [$N \cdot s / m$]
- f_V : Variable viscous friction coefficient depends on the speed and forces applied to the cylinder [$N \cdot s / m$]
- Kv : Valve flow coefficient [$m^3 / s \cdot \sqrt{Pa}$]
- K : Stiffness [N/m]
- F : Force [N]
- F_{at} : Friction force [N]
- F_U : Useful force [N]
- ω_n : Natural frequency [*rad* or *Hz*]
- K_{I} : Controller integral gain [dimensionless]
- K_s : Position transducer gain [V/m]
- K_{z} : Controller proportional gain [dimensionless]
- M : Mass [kg]
- ρ : Mass density [kg / m³]
- β : Isothermal bulk modulus [*Pa*]
- β_e : Effective bulk modulus [*Pa*]

- x : Position [m]
- *p* : Pressure [*bar* or *Pa*]
- p_C : Load pressure [*bar* or *Pa*]
- p_T : Return pressure [*bar* or *Pa*]
- p_s : Hydraulic supply pressure [*bar* or *Pa*]
- p_P : Pneumatic supply pressure [*bar* or *Pa*]
- p_A : Pressure in chamber A [*bar* or *Pa*]
- p_B : Pressure in chamber B [*bar* or *Pa*]
- ζ : Damping ratio [dimensionless]
- t_s : Settling time [s]
- U : Electrical tension [V]
- q_V : Flow rate [lpm or m^3/s]
- q_{VC} : Control flow rate [*lpm* or m^3/s]
- v : Speed [m/s]
- HPU : Hydraulic power unit.
- **PHP** : Proportional Hydraulic Platform
- PI : Proportional Integral Controller
- PLC : Programmable Logical Controller