A Park - like transform for fluid power systems: application to pneumatic stiffness control

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Context & problematics

- Modeling and control of Fluid Power systems is usually considered as a difficult task:
  - **multiphysic**: mechanics, fluid dynamics, thermodynamics, electrical eng., …
  - highly non linear behavior

more difficult to control than electrical drives?  

NO
Differences electric and fluid power drives (1/3)

Electric drives

- Desired command
- Angular position

Electric Power

High bandwidth
- Lot of sensors (current, voltage)
- Complex control algorithm embedded (large computation capacity)
- But everything is included !!!

Load

Low bandwidth
- High reflected inertia
- Complex friction phenomena
- Can be considered part of the load (disturbance)

The whole package is on-the-shelves !!!
Hydraulic drives

- Low bandwidth
  - ~ no sensor (mech/elec feedback)
  - each design is different
  - safety component have to be added

- High bandwidth
  - low inertia
  - nearly a integrator

Not user friendly

Help yourself !!!!
Other architectures of hydraulic drives

- **Mechanic Power**
  - Low bandwidth

- **Electric Power**
  - High bandwidth
  - Not user friendly

- **Fluid Power drives**
  - High bandwidth
Pneumatic drives

- High bandwidth
  - ~ no sensor (mech/elec feedback)
  - low efficiency
  - each design is different
  - safety component to be added

- Low bandwidth
  - low inertia
  - low pressure dynamic

Not user friendly

Help yourself !!!!
Why electrical drive are so user-friendly?

- **Servo/Prop. Hydraulic Act.**
- **EHA/ Displacement pump**
- **Nearly no computation capacity embedded**
  - **Not Plug & Play**
- **Servo Pneumatic Act.**
- **EHA/ variable speed**
- **Electric MA**

The intelligence is in the box

Plug & Play

User-friendly control
How to make the control easier?

- The physical variables are not always the best for control purposes:
  
  - **On mechanical side**
    
    \[
    \begin{align*}
    \frac{dv}{dt} &= \frac{S.(p_P - p_N) - F_{frot}(v) - F_{ext}}{M} \\
    \frac{dy}{dt} &= v
    \end{align*}
    \]
    
    \[F_{pneu} = S.(p_P - p_N)\]
    
    - **On fluid side (pneumatic)**
      
      \[
      \begin{align*}
      \frac{dp_P}{dt} &= \frac{k.r.T}{V_P} \left(q_{mP} - \frac{S}{r.T}p_P.v\right) \\
      \frac{dp_N}{dt} &= \frac{k.r.T}{V_N} \left(q_{mN} + \frac{S}{r.T}p_N.v\right)
      \end{align*}
      \]
      
      2 commands:
      
      \[
      \begin{align*}
      &q_{mP}(u_P, p_P) \\
      &q_{mN}(u_N, p_N)
      \end{align*}
      \]
      
      \[
      \begin{align*}
      u_N &= \psi^{-1}(q_{mN}, p_N) \\
      u_P &= \psi^{-1}(q_{mP}, p_P)
      \end{align*}
      \]
The AT transform (1/2)

- 2 different phenomena
  - The differential pressurization:
    \[
    \frac{d\Delta p}{dt} = \frac{dp_P}{dt} - \frac{dp_N}{dt}
    \]
  - The symmetric pressurization:
    \[
    \frac{dp_T}{dt} = \frac{1}{2} \left( \frac{dp_P}{dt} + \frac{dp_N}{dt} \right)
    \]

- Let us consider the following coordinate transformation:

  \[
  \begin{bmatrix}
  q_{mA} \\
  q_{mT} \\
  q_{mN}
  \end{bmatrix} = \Lambda(y) \cdot \begin{bmatrix}
  q_{mP} \\
  q_{mN}
  \end{bmatrix}
  \]

  \[
  \Lambda(y) = \begin{bmatrix}
  \frac{1}{V_P(y)} & \frac{1}{V_N(y)} \\
  \frac{1}{V_P(y)} & \frac{1}{V_N(y)}
  \end{bmatrix}
  \]

  \[
  \begin{cases}
  q_{mP} & \rightarrow q_{mA} = \text{active flow} \\
  q_{mN} & \rightarrow q_{mT} = \text{pressurization flow}
  \end{cases}
  \]

  * Note: this transformation can easily be extended to non-symmetric cylinder
The AT transform (2/2)

Make the really interesting variables appear in the equations

\[
\begin{align*}
\frac{dp_P}{dt} &= \frac{k.r.T}{V_P} \cdot (q_{m_P} - \frac{S}{r.T}p_P.v) \\
\frac{dp_N}{dt} &= \frac{k.r.T}{V_N} \cdot (q_{m_N} + \frac{S}{r.T}p_N.v)
\end{align*}
\]

\[
\begin{bmatrix}
q_{mA} \\
q_{mT}
\end{bmatrix} = \Lambda(y). \begin{bmatrix}
q_{mP} \\
q_{mN}
\end{bmatrix}
\]

\[
F_{pneu} = S(P_P - P_N) = S \Delta p
\]

\[
P_T = \frac{P_N + P_P}{2}
\]

\[
\frac{dp_T}{dt} = -\frac{k.S.v}{2} \cdot \left( \frac{p_P}{V_P} - \frac{p_N}{V_N} \right) + \frac{k.r.T}{2.V_0} \cdot q_{mT}
\]

\[
\frac{d\Delta p}{dt} = -k.S.v \cdot \left( \frac{p_P}{V_P} + \frac{p_N}{V_N} \right) + \frac{k.r.T}{V_0} \cdot q_{mA}
\]
The AT transform (2/2)

- Make the really interesting variables appear in the equations

\[
\begin{align*}
\frac{dy}{dt} &= v \\
\frac{dv}{dt} &= -b \cdot v - F_{\text{sec}}(v) + \frac{F_{\text{pneu}}}{M} \\
\frac{dF_{\text{pneu}}}{dt} &= \frac{A_1 \cdot v \cdot y \cdot F_{\text{pneu}} - A_2 \cdot v \cdot p_T}{V_P \cdot V_N} + B_1 q_m A \\
\frac{dp_T}{dt} &= -\frac{A_3 \cdot v \cdot F_{\text{pneu}} + A_4 \cdot v \cdot y \cdot p_T}{V_P \cdot V_N} + B_2 q_m T
\end{align*}
\]

Decoupling Force generation from Pressurization
The AT transform (2/2)

- Make the really interesting variables appear in the equations

\[
\begin{align*}
\frac{dy}{dt} &= v \\
\frac{dv}{dt} &= \frac{-b.v - F_{sec}(v) + F_{pneu}}{M} \\
\frac{dF_{pneu}}{dt} &= \frac{A_1.v.y.F_{pneu} - A_2.v.p_T}{V_P.V_N} + B_1 q_{mA} \\
\frac{dp_T}{dt} &= \frac{-A_3.v.F_{pneu} + A_4.v.y.p_T}{V_P.V_N} + B_2 q_{mT}
\end{align*}
\]

- \( F_{pneu} = S.(p_P - p_N) \)
- Speed
- Displacement
- Active flow
- Pressurization flow

\( P_T \sim \text{Actuator Stiffness} \)
Experimental validation

- Open-loop: variation on $q_{mT}$

$$\frac{dp_T}{dt} = -\frac{k.S.v}{2} \left( \frac{p_P}{V_P} - \frac{p_N}{V_N} \right) + \frac{k.r.T}{2.V_0} q_{mT}$$

$\Delta P = 0$

$\frac{d\Delta p}{dt} = -k.S.v \left( \frac{p_P}{V_P} + \frac{p_N}{V_N} \right) + \frac{k.r.T}{V_0} q_{mA}$

Variation of 4 bars

Variation of 0.7 bars
Experimental validation

- Open-loop: variation on $q_{mA}$

$$\frac{dp_T}{dt} = -\frac{k.S.v}{2} \left( \frac{p_P}{V_P} - \frac{p_N}{V_N} \right) + \frac{k.r.T}{2.V_0} q_{mT}$$

Virtual flow rates [g/s]

$q_{mT} = 0$

$q_{mA}$

$$\frac{d\Delta p}{dt} = -k.S.v \left( \frac{p_P}{V_P} + \frac{p_N}{V_N} \right) + \frac{k.r.T}{V_0} q_{mA}$$

Variation < 0.05 bar

Variation max of 0.8 bars
AT Transform vs. Park Transform

**AT Transform**
- change of variables for control purpose:
  \[
  \begin{bmatrix}
  qm_A \\
  qm_T
  \end{bmatrix} = \Lambda(y) \cdot \begin{bmatrix}
  qm_P \\
  qm_N
  \end{bmatrix}
  \]
- original flows of power modulator change in:
  - **virtual active flow** \( qm_A \)
  - **virtual presurization flow** \( qm_T \)

**Park Transform**
- change of variables for control purpose:
  \[
  \begin{bmatrix}
  V_d \\
  V_q
  \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix}
  \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\
  -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right)
  \end{bmatrix} \cdot \begin{bmatrix}
  V_a \\
  V_b \\
  V_c
  \end{bmatrix}
  \]
- original voltages of power modulator change in:
  - **virtual voltage** \( V_d \)
  - **virtual voltage** \( V_q \)

**Displacement and force generation**
- **virtual active flow** \( qm_A \)
- **virtual voltage** \( V_d \)

**Torque control**
- **virtual voltage** \( V_q \)

**Mean pressure control**
- **virtual presurization flow** \( qm_T \)

**Magnetic flux control**
- **Actuator stiffness**
Applications of the AT Transform

- Trajectory control ($Y-P_T$ control)
- Energy saving ($Y-P_{Topti}$ control)
- Displacement / Stiffness ($Y-K$ control)
- Position observer (at 0 speed)
- Mono-distributor
Application of the AT Transform

- **Active flow**

  - \( q_{mA} \)
  
  - Pressure difference: \( F_{pneu} = S(p_P - p_N) \)
  
  - Speed: \( \int \)
  
  - Displacement: \( \int \)

- **Pressurization flow**

  - \( q_{mT} \)
  
  - Pressurization control: (Y-P\(_T\) control)
  
  - Energy saving: (Y-P\(_{Topti}\) control)
  
  - Stiffness: (Y-K control)
  
  - Position observer: (at 0 speed)
  
  - Mono-distributor
Trajectory control (Y-P_T control)

Control synthesis: Backstepping

Same results to what was achieved with other control techniques
Energy saving (Y-P_{Topti} control)

Control synthesis:
- $q_{mA}$ is imposed (y trajectory)
- minimize

\[ C(t) = |q_{mP}(t)| + |q_{mN}(t)| \]

with

\[ q_{mA} = V_0 \cdot \left( \frac{q_{mP}}{V_P} - \frac{q_{mN}}{V_N} \right) \]

If $V_N$ smaller than $V_P$, less flow is required to produce $q_{mA}$ if $q_{mN}$ is used
If $V_P$ smaller than $V_N$, less flow is required to produce $q_{mA}$ if $q_{mP}$ is used

\[
\begin{aligned}
q_{mP} &= 0 ; \ q_{mN} = -\frac{V_N}{V_0} \cdot q_{mA} \text{ if } y > 0, \\
q_{mP} &= \frac{V_P}{V_0} \cdot q_{mA} ; \ q_{mN} = 0 \text{ if } y < 0, \\
q_{mP} &= -q_{mN} = \frac{q_{mA}}{2} \text{ if } y = 0
\end{aligned}
\]

as

\[ q_{mT} = V_0 \cdot \left( \frac{q_{mP}}{V_P} + \frac{q_{mN}}{V_N} \right) \]

Optimal control is obtained for:

\[ q_{mT} = -q_{mA} \cdot sgn(y) \]
Stiffness control:
- reject or not disturbances in open-loop (do not required fast closed loop)
- variable compliance

Control synthesis:
- \( q_{mA} \) is imposed \((y\) trajectory\)
- Actuator stiffness 
\[
K_{pneu} = \left( \frac{pP}{V_P(y)} + \frac{pN}{V_N(y)} \right) . k . S^2
\]

\[
\begin{align*}
\frac{dy}{dt} &= v \\
\frac{dv}{dt} &= -b.v - F_{sec}(v) + F_{pneu} \\
\frac{dF_{pneu}}{dt} &= -K_{pneu}v + B_1 . q_{mA} \\
\frac{dK_{pneu}}{dt} &= \frac{A_5 . K_{pneu} . y . v - A_6 . F_{pneu} . v - B_4 . y . q_{mA} + B_5 . q_{mT}}{V_N(y) . V_P(y)}
\end{align*}
\]

natural behavior of the actuator to come back to its equilibrium point
**Stiffness (Y-K control)**

- **Control synthesis : Backstepping**

- Close loop stiffness

- **Desired** $K_{pneu}$
  - $K_{pneu} = 1.5 \times 10^5$ N/m
  - $K_{pneu} = 2.5 \times 10^5$ N/m
  - $K_{pneu} = 3.5 \times 10^5$ N/m

- **Measured** $K_{pneu}$

- **Displacement** $y$

- **Load** [N]

- **Virtual flow rates** [g/s]
  - $K_{pneu} = 1.5 \times 10^5$ N/m
  - $K_{pneu} = 2.5 \times 10^5$ N/m
  - $K_{pneu} = 3.5 \times 10^5$ N/m

- Static error decreasing with $K_{pneu}$ at impact

FPNI 2016, 26-28 oct. 2016, Florianopolis, Brazil
Position observer at standstill (v=0):

\[
\begin{align*}
\frac{dy}{dt} &= 0 \\
\frac{dv}{dt} &= -F_{\text{sec}}(v = 0) + F_{\text{pneu}} \\
\frac{dF_{\text{pneu}}}{dt} &= B_1 q_{mA} \\
\frac{dp_T}{dt} &= B_2 q_{mT}
\end{align*}
\]

- \( y, q_{mA}, q_{mT} \) not known

From pressure sensors only:

- \( F_{\text{pneu}} \) and \( P_T \) can be calculated,
- the full state can be obtained by differentiation:

Procedure

- change slightly \( q_{mT} \) with \( q_{mA} = 0 \)

theoretically \( F_{\text{pneu}} \) do not change (or < dry friction)

the piston do not move (v=0)
Observer synthesis: Sliding mode

- 5 Hz Sinusoidal trajectory on $P_T$

- Force $F_{\text{pneu}}$ measured vs time [s]
- Displacement $y$ vs time [s]
- Derivative of $F_{\text{pneu}}$ vs time [s]

---

Estimated position $y$

Real position $y_m$

FPNI 2016, 26-28 oct. 2016, Florianopolis, Brazil
Conclusion

- Active flow
  - $q_m A$
  - Pressure difference
    \[ F_{pneu} = S(p_P - p_N) \]
  - \( \int \) Speed
  - \( \int \) Displacement

- Pressurization flow
  - $q_m T$
  - Pressurization control
    (Y-$P_T$ control)
  - Energy saving
    (Y-$P_{Topti}$ control)
  - Stiffness
    (Y-$K$ control)
  - Position observer
    (at standstill)
  - Mono-distributor

What else?
Thanks a lot for your kind attention...

References:
